ON THE EFFECTIVE LIGHT-CONE QCD-HAMILTONIAN

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Taking the effective interaction between a quark and an anti-quark from previous work, the dependendence on a regularization scale is removed in line with the renormalization group. In order to emphasize the essential point, the full spinor interaction is replaced by a model which includes only the Coulomb and the hyperfine interaction. By adjusting the effective quark masses, the only free parameters of the theory, the mass and the size of the pion are reproduced, as well as the mass of all other pseudo-scalar mesons. Estimates for the vector mesons are close to the empirical values. The model exposes screening rather than strict confinement. The ionization thresholds are in general much larger than the pion mass.

1 Introduction

The bound-state problem in a field theory particularly gauge field theory is a very difficult many-body problem and has not been solved thus far. But perhaps an exact solution is not necessary for reconciling the so succesful constituent quark models with a covariant theory such as QCD and for being able to compute observables like the structure functions and distribution amplitudes from an underlying theory. Perhaps it suffices to have approximate or QCD-inspired solutions which are not wrong from the outset. The present work is still ongoing and contributes to these questions. Perhaps one begins to understand the essential point.

2 A QCD-inspired effective interaction

The light-cone approach to the bound state problem aims at diagonalizing the invariant mass-squared operator.[1] Its matrix elements are obtained directly from the gauge field Lagrangian in the light-cone gauge and are explicitly tabulated.[1] The many-body aspect of the problem was reduced recently, by the method of iterated resolvents,[2] to the problem of phrasing an effective Hamiltonian which acts only in the Fock space of one quark and one anti-quark. The reduction is not exact but rests on certain simplifying assumptions which can be checked only *a posteriori*. Quite cautiously, one should therefore speak of a 'QCD-inspired' effective Hamiltonian. Important is that both the effective and the full Hamiltonian have in principle the same eigenvalue spectrum and that the Fock space is systematically reduced to the $q\bar{q}$-space. Important is also that the Fock-space is not truncated, that all Lagrangian symmetries

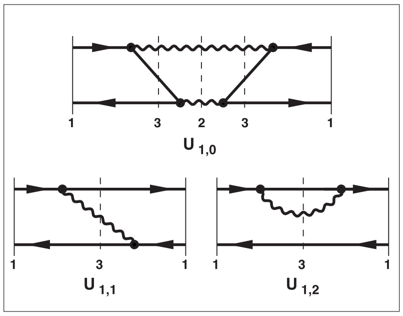

Figure 1: The three graphs of the effective interaction in the $q\bar{q}$-space according to[2]. The upper graph illustrates an effective interaction due to a two gluon annihilation. For flavor-off-diagonal mesons its amplitude is kinematically suppressed. The lower two graphs correspond to an effective one gluon exchange. – The effective interaction scatters a quark with four momentum and helicity (k_1, λ_1) into (k_1', λ_1'), and correspondingly the anti-quark from (k_2, λ_2) into (k_2', λ_2').

are preserved, and that the higher Fock-space amplitudes can be retrieved systematically from the $q\bar{q}$-projection once that is available.

The effective interaction has several contributions which are illustrated in Fig. 1. The present work is restricted to flavor-off-diagonal mesons, to mesons where quark and anti-quark have different flavors. Then the effective interaction due to the annihilation of two gluons is kinematically suppressed, and the final (one-body) integral equation in the $q\bar{q}$-space is governed by an effective one gluon exchange, *i.e.*

$$M^2 \langle x, \vec{k}_\perp; \lambda_1, \lambda_2 | \psi \rangle = \left[\frac{\overline{m}_1^2 + \vec{k}_\perp^{\,2}}{x} + \frac{\overline{m}_2^2 + \vec{k}_\perp^{\,2}}{1-x} \right] \langle x, \vec{k}_\perp; \lambda_1, \lambda_2 | \psi \rangle$$
$$- \frac{1}{4\pi^2} \sum_{\lambda_q', \lambda_2'} \int \frac{dx' d^2 \vec{k}_\perp'}{\sqrt{x(1-x)x'(1-x')}} R(x', \vec{k}_\perp'; \Lambda) \times$$
$$\times \frac{4}{3} \frac{\overline{\alpha}(Q;\Lambda)}{Q^2} \langle \lambda_1, \lambda_2 | S | \lambda_1', \lambda_2' \rangle \, \langle x', \vec{k}_\perp'; \lambda_1', \lambda_2' | \psi \rangle. \qquad (1)$$

Here, M^2 is the invariant-mass squared eigenvalue and $\langle x, \vec{k}_\perp; \lambda_1, \lambda_2 | \psi \rangle$ the associated eigenfunction. It is the probability amplitude for finding a $q\bar{q}$-Fock state in which the quark has longitudinal momentum fraction x and transversal momentum $\vec{k}_\perp$ and the anti-quark correspondingly $1-x$ and $-\vec{k}_\perp$, and where the respective helicities are λ_1 and λ_2. The mean four-momentum transfers of the quarks and the spinor factor are respectively defined by

$$Q^2 = -\frac{1}{2} \left[(k_1 - k_1')^2 + (k_2 - k_2')^2 \right], \qquad (2)$$
$$S = \langle \lambda_1, \lambda_2 | S | \lambda_1', \lambda_2' \rangle = \left[\overline{u}(k_1, \lambda_1) \gamma^\mu u(k_1', \lambda_1') \right] \left[\overline{v}(k_2', \lambda_2') \gamma_\mu v(k_2, \lambda_2) \right]. \qquad (3)$$

The effective quark masses $\overline{m}_{1,2}$ and the effective coupling constant $\overline{\alpha}(Q;\Lambda)$ depend both, in general, on a regularization scale Λ. The regulator function

$R(x', \vec{k}'_\perp; \Lambda)$ restricts the range of integration and in general is a function of the same Λ, see also below.

The current work is restricted to the lowest order of approximation (LOA) where $\overline{m} = m$ and $\overline{\alpha}(Q; \Lambda) = \alpha$. The expressions for the next-to-lowest order (NLO) can be found elsewhere.[2] One has a fair confidence into the validity of Eq.(1), since the alternative method of Hamiltonian flow [3] leads to the very same equation as in LOA.[4] The same equation had also been obtained prior to that,[5,6] with however less stringent arguments. The work of Wilson *et al.* [7] had the same emphasis, but that it did not lead to the same formulas, see also Refs.[8,9].

3 The crucial point in a simple model

Why is a regularization necessary at all?

In light-cone parametrization, the quark is at rest relative to the anti-quark when $\vec{k}_\perp = 0$ and $x = \overline{x} \equiv \overline{m}_1/(\overline{m}_1 + \overline{m}_2)$. For very small deviations from these 'equilibrium values' the spinor factor is diagonal in the helicities, $\langle \lambda_1, \lambda_2 | S | \lambda'_1, \lambda'_2 \rangle \sim 4\overline{m}_1 \overline{m}_2\, \delta_{\lambda_1, \lambda'_1}\, \delta_{\lambda_2, \lambda'_2}$, as can be veryfied from an explicit presentation of S.[6] For very large deviations, particularly for $\vec{k}'^2_\perp \gg \vec{k}^2_\perp$, holds $Q^2 \simeq \vec{k}'^2_\perp$ and $\langle \uparrow\downarrow |S| \uparrow\downarrow \rangle \simeq 2\vec{k}'^2_\perp$. The ratio is then a simple dimensionless number, $S/Q^2 = 2$, independent of $\vec{k}'_\perp$. Both extremes are combined here by

$$\frac{\langle \uparrow\downarrow |S| \uparrow\downarrow \rangle}{Q}^2 = \frac{4\overline{m}_1 \overline{m}_2}{Q^2} + 2, \tag{4}$$

which is considered as a *model of the interaction* in singlet states, dropping the less important spin-orbit interaction.

Unfortunately, I have not realized earlier [5,6] that the innocent and finite number '2' in Eq.(4) has the same orign as the familiar divergencies in gauge field theory. The latter arise, typically, when bilinar products of the elementatry Dirac interaction like in S divided by some energy denominator like Q^2 are integrated over all phase space.

It was clear from the outset that the many-body Hamiltonian has to be regularized.[5] Since fancy methods like dimensional regularization are not applicable, Fock-space regularization was considered to be sufficient.[1] In practice, it has lead to difficulties and was was replaced by vertex regularization,[2] where the elementary Dirac interaction at a vertex was endorsed with kind of a form factor $F(\Lambda)$, *i.e.*

$$[\overline{u}(1)\gamma^\mu u(2)]\, \epsilon_\mu(3) \longrightarrow [\overline{u}(1)\gamma^\mu u(2)]\, \epsilon_\mu(3)\, F(k_1, k_2, k_3; \Lambda). \tag{5}$$

Requiring that the off-shell mass of the two scattered particles $M_0^2 = (k_2+k_3)^2$ is limited, F becomes the step function $F(k_1,k_2,k_3;\Lambda) = \Theta(M_0^2 - \Lambda^2)$. In the absolute squares of the effective interaction, as in Eq.(1), it appears as a regulator function R as a consequence of regulating the theory from the outset.

4 Rewriting the integral equation in conventional momenta

Unpleasant is that x and $\vec{k}_\perp$ have different ranges ($0 \le x \le 1,\ -\infty \le k_x \le \infty$). Instead of x a k_z ($-\infty \le k_z \le \infty$) can be introduced by the transform [10]

$$x(k_z) = \frac{E_1 + k_z}{E_1 + E_2}, \qquad \text{with } E_{1,2} = \sqrt{\overline{m}_{1,2}^2 + \vec{k}_\perp^{\,2} + k_z^2}. \tag{6}$$

The Jacobian of the transformation can be cast into the form

$$\frac{dx}{x(1-x)} = \frac{1}{A(k)}\frac{dk_z}{m_r}, \qquad \text{with } A(k) = \frac{1}{m_r}\frac{E_1E_2}{E_1+E_2}, \tag{7}$$

$$\text{and} \qquad \frac{1}{m_r} = \frac{1}{\overline{m}_1} + \frac{1}{\overline{m}_2}, \qquad m_s = \overline{m}_1 + \overline{m}_2. \tag{8}$$

In the same convention the kinetic energy becomes

$$T(k) = \frac{\overline{m}_1^2 + \vec{k}_\perp^{\,2}}{x} + \frac{\overline{m}_2^2 + \vec{k}_\perp^{\,2}}{1-x} - m_s^2 \quad \equiv \quad C(k)\,\vec{k}^2, \tag{9}$$

$$\text{with} \quad C(k) = (E_1 + \overline{m}_1 + E_2 + \overline{m}_2)\left(\frac{1}{E_1+\overline{m}_1} + \frac{1}{E_2+\overline{m}_2}\right), \tag{10}$$

without approximation. If one substitutes the wave function according to

$$\psi(x,\vec{k}_\perp) = \sqrt{\frac{A(x,\vec{k}_\perp)}{x(1-x)}}\;\phi(x,\vec{k}_\perp) \tag{11}$$

and drops explicit reference to the helicities, one arrives at an equation,

$$\left[M^2 - m_s^2 - C(k)\vec{k}^2\right]\phi(\vec{k}) = -\frac{1}{4\pi^2 m_r}\int \frac{d^3\vec{k}'\; R}{\sqrt{A(\vec{k})A(\vec{k}')}}\;\frac{4}{3}\frac{\overline{\alpha}S}{Q^2}\,\phi(\vec{k}'), \tag{12}$$

which is identical with Eq.(1), but which looks like one with usual 3-momenta.

5 Interpretation in configuration space

The main reason for introducing conventional 3-momenta is interpretation. It is more transparent in configuration space. Since the various factors $A(\vec{k})$

with their square-roots prevent straight-forward Fourier transforms, the non-relativistic approximation ($\vec{k}'^2 \ll \overline{m}_{1,2}^2$) is applied consistently. Using

$$A(k) = 1, \qquad C(k) = \frac{m_s}{m_r}, \qquad Q^2 = (\vec{k} - \vec{k}')^2, \tag{13}$$

the model interaction of Eq.(4), and in addition the regulator $R = 1$, Eq.(12) leads to a local Schrödinger equation

$$\left[M^2 - m_s^2 - \frac{m_s}{m_r}\vec{k}^2\right]\phi(\vec{k}) = -\frac{4\alpha}{3\pi^2}\int d^3\vec{k}' \left(\frac{m_s}{Q^2} + \frac{1}{2m_r}\right)\phi(\vec{k}'), \tag{14}$$

$$\left[M^2 - m_s^2 - \frac{m_s}{m_r}\vec{p}^2\right]\psi(\vec{r}) = 2m_s V(r)\psi(\vec{r}), \quad \text{with } \vec{p} \equiv i\vec{\nabla}. \tag{15}$$

The potential $V(r)$ bears great similarity with $V_{\text{hf-s}}(r)$, the hyperfine interaction in the singlet channel found for hydrogen in all textbooks, *i.e.*

$$V(r) = -\frac{4}{3}\alpha\left(\frac{1}{r} + 2\ \frac{\pi}{m_r m_s}\delta^{(3)}(\vec{r})\right), \tag{16}$$

$$V_{\text{hf-s}}(r) = -\ \alpha\left(\frac{1}{r} + g_p\frac{\pi}{m_e m_p}\delta^{(3)}(\vec{r})\right). \tag{17}$$

The strange '2' in Eq.(4) finds its explanation as the gyromagnetic ratio for a fermion, with $g_p = 2$. But Eq.(16) has no solution! A Dirac-delta function is no proper function and must be regulated, for instance by a Yukawa-potential

$$V(r) = -\frac{4\alpha}{3}\left(\frac{1}{r} + \frac{\mu^2}{m_r m_s}\,\frac{e^{-\mu r}}{r}\right). \tag{18}$$

Important is that the delta ($\int d^3\vec{r}\,\delta^{(3)}(\vec{r}) = 1$) and the Yukawa have the same strength $\int d^3\vec{r}\,[\mu^2\exp(-\mu r)/(2\pi r)] = 1$. Transforming back to momentum space gives

$$\left[M^2 - m_s^2 - \frac{m_s}{m_r}\vec{k}^2\right]\phi(\vec{k}) = -\frac{4\alpha}{3\pi^2}\int d^3\vec{k}' \left(\frac{m_s}{Q^2} + \frac{1}{2m_r}\frac{\mu^2}{\mu^2 + Q^2}\right)\phi(\vec{k}'). \tag{19}$$

Obviously, replacing in configuration space the Dirac-delta function with a Yukawa potential corresponds to introducing in momentum space the regulator function $R = \mu^2/(\mu^2 + Q^2)$.

The regularization of a Dirac-delta function is an old theme of nuclear physics in the context of pairing theory.[11] It was the point of orign for the similarity transform,[12,13] and was investigated recently,[14] again.

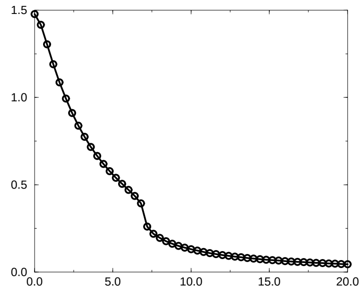

Figure 2: Coupling constant α versus regularization scale μ in units of 350 MeV.

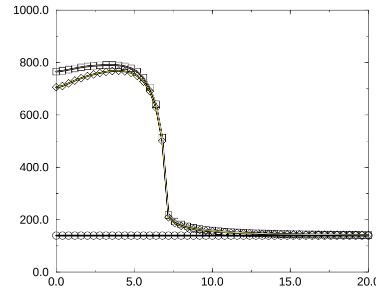

Figure 3: The spectrum of the first three singlet-s states in MeV versus regularization scale μ in units of 350 MeV.

6 Renormalization

The eigen values of the integral equation Eq.(19) depend now on a regularization scale μ. In line with the current understanding of field theory one replaces α and m by functions of μ, such that every eigenvalue is stationary against small variations of α, m and μ, *i.e.* $dM_n^2/d\mu = 0$! The general procedure, however, is not possible here, since α and μ appear in Eq.(19) in the typical combination $\alpha\mu^2/(\mu^2+Q^2)$. One therefore proceeds by looking for a function α_μ such, that the *calculated* mass of the pion, for example, agrees with the empirical value. The so obtained function α_μ is considered universal. The actual value of μ can then be fixed by a second requirement.

7 The pion

For carrying out, in practice, the programme of Sec. 6 one needs an efficient tool for solving Eq.(19). Such one has been developed recently,[15] restricted however for simplicity to spherically symmetric s-states. Fixing the up and the down mass to $\overline{m}_u = \overline{m}_d = 1.16$ (the unit of mass is chosen here as 350 MeV/c^2), the calculated pion mass is required to agree with the experimental value [16] to within eight digits. For every value of μ one obtains thus an α_μ which is displayed in Fig. 2. The even greater surprise came when investigating the spectrum of the pion along the so obtained α_μ, as given in Fig. 2. The lowest state, the π^+, stays nailed fixed to the empirical value, while the second and the third (as well as the higher ones) have an extremum at $\mu \simeq 3.8$. Such an extremum was not really expected, but in view of the renormalization group

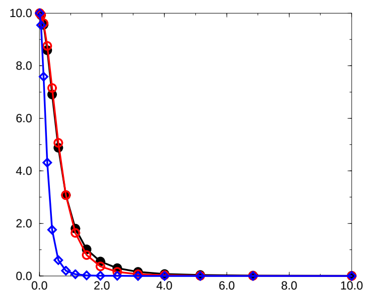

Figure 4: The pion wave function $\Phi(p)$ is plotted versus $p/1.552$ (filled circles) in an arbitrary normalization. The diamonds denote a Coulomb wave function with the same α and the open circles an approximate wave function, see text.

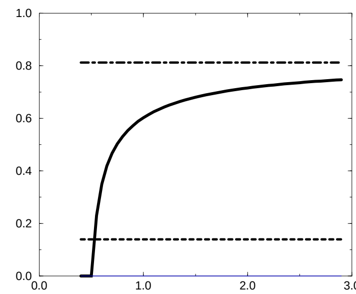

Figure 5: The relativistic potential energy $W(r)$ for the pion is plotted in GeV versus the radius in fm (solid line). The pion mass is indicated by the dashed, and the ionization threshold by the dashed-dotted line.

is useful: μ determines itsself from the solution!

For suffiently large values of μ, say for $\mu \gg 10$, see Fig. 3, the coupling constant decreases strongly and the spectrum becomes more and more the familiar Bohr spectrum with a small hyperfine shift. Obviously, it can be calculated perturbatively, indeed, for sufficiently small α.

Having fixed now the regularization scale $\mu \simeq \mu_0 = 3.8$ gives the coupling constant $\alpha \simeq \alpha_{\mu_0} = 0.6904$, see also Fig. 2. The wave function of the pion is plotted in Fig. 4. By a fit to an approximate expression $\Phi_a(p) = 10/(1+p^2/p_a^2)$, a width $p_a = 1.471$ is obtained which is much larger than $p_c = \frac{4}{3}\alpha m_r = 0.5339$, the Bohr momentum of the Coulomb wave function with the same α. Having the wave function, one could, in principle, calculate the form factor and the distribution amplitude according to the exact expressions.[1] This is however not a trivial undertaking, and, instead of, the above approximate expression was Fourier transformed to $\psi(\vec{r}) \sim e^{-p_a r}$. The calculated root-mean-sqare radius of the pion becomes then

$$< r^2 >^{\frac{1}{2}} = \frac{\sqrt{3}}{p_a} = 0.663 \text{ fm}. \tag{20}$$

The good agreement with the experimental value $< r^2 >^{\frac{1}{2}}_{\pi} = 0.657$ fm[16] is actually the result of fitting the quark masses $\overline{m}_u = \overline{m}_u = 406$ MeV, see above. This exhausts all freedom in choosing the parameters of Eq.(19).

One should emphasize that the Yukawa potential in Eq.(19) at the scale μ_0 pulls down essentially only one state. The others are left rather unperturbed at their Bohr values, see Fig. 3. The analogy with the pairing model in early nuclear physics is more than obvious.[11] In a future and more rigorous solution of the full Eq.(12) one expects that the excited states can be disentangled into almost degenerate singlets and triplets, which in turn can be interpreted as an excitation of the pseudo-scalar pion, or the ground state of a vector meson, respectively. In any case, the first excited state of the simple model correlates very well with the mass of the ρ^+ and the other vector mesons, see below.

8 On the question of confinement

The question of confinement is closely related to the potential $V(r)$, but the relation is subtle. One of the great advantages of light-cone quantization is the additivity of the free part and the interaction. Because of the dimensions of invariant mass squares the relation of potential and kinetic energy is somewhat hidden. In analogy to a classical Hamiltonian, the invariant mass-squared operator M^2 in Eq.(19) is interpreted as the square of a position and momentum dependent mass

$$M(r,p) = m_s \sqrt{1 + \frac{p^2}{m_s m_r} + \frac{2}{m_s} V(r)}. \tag{21}$$

For $V(r) \ll m_s$ and $p^2 \ll m_s m_r$ it could be expanded as

$$\begin{aligned} M(r,p) &\simeq m_s + \frac{p^2}{2m_r} + V(r), \\ &\simeq \text{rest mass} + \text{kinetic energy} + \text{potential energy}, \end{aligned}$$

These conditions, however, can not be satisfied everywhere. I therefore propose to introduce a *relativistic potential energy* by $W(r) \equiv M(r, p = 0)$. This function is plotted in Fig. 5. It vanishes at $r_0 = 0.5194$ fm and the classical turning point is $r_t = 0.5300$ fm, in agreement with Eq.(20). Since the sum of the quark masses is 812 MeV, the ionization threshold occurs at 542 MeV: In order to liberate the quarks in the pion, one has to invest more than three times the rest energy of a pion.

9 The meson masses

The remaining parameters of the theory, the masses of the strange, charm and bottom quark can then be determined by reproducing the masses of the pseudo-scalar mesons $K,^-$ D^0 and $B,^-$ respectively. This gives $\overline{m}_s = 508$, $\overline{m}_c = 1666$ and $\overline{m}_b = 5054$ MeV. The remaining off-diagonal pseudo-scalar

	$\overline{u}$	$\overline{d}$	$\overline{s}$	$\overline{c}$	$\overline{b}$	$\overline{u}$	$\overline{d}$	$\overline{s}$	$\overline{c}$	$\overline{b}$
u		768	892	2007	5325		768	871	2030	5418
d	140		896	2010	5325	140		871	2030	5418
s	494	498		2110	—	494	494		2124	5510
c	1865	1869	1969		—	1865	1865	1929		6580
b	5278	5279	5375	—		5279	5279	5338	6114	

Table 1: *Left:* Experimental masses of the flavor-off-diagonal physical mesons in MeV. Scalar mesons are given in the lower, vector mesons in the upper triangle. – *Right:* The calculated mass eigenstates of QCD in MeV. Singlet-1s states are given in the lower, singlet-2s states in the upper triangle.

mesons are then calculated straightforwardly and compiled in Table 1. In view of the simplicity of the model, the agreement with the empirical values is remarkable, indeed, and at least as good as for any other model. The strong correlation of the first excited state with the vector mesons was mentioned.

10 Summary and some conclusions

The very difficult problem of solving the many-body Hamiltonian for QCD is replaced here by the problem of solving an effective QCD-inspired Hamiltonian. It is obtained from the QCD-Lagrangian in the light-cone gauge by the method of iterated resolvents or the Hamiltonian flow equations in the lowest non-trivial approximation. In order to avoid the numerical complexities of the full solution in light-cone coordinates, this effective interaction was stripped-off of almost all ingredients except the Coulomb and the hyperfine interaction, which are analyzed in a non-relativistic treatment with only 3-momenta. It is argued why the hyperfine interaction in the singlet channel needs to be regulated like the usual divergencies in gauge field theory.

It was courageous to apply such a simple model to the mystery particle of QCD, to the pion, but it turns out that both the pion's mass and size are obtained without vacuum condensates of any kind, just by adjusting the canonical parameters of a gauge field theory, the coupling constant and the quark masses. The necessary regularization constant determines itself by the theory, in line with the current interpretation of the renormalization group. For the pion, as well as for the other mesons, one gets thus for the first time a QCD-inspired light-cone wave function by which one can compute the form factor by the available exact formulas, as well as the higher Fock space amplitudes.

The present approach exposes screening rather than confinement. The

removal energy of a quark from a meson is higher than the pion's mass. A bare quark is thus unlikely to be observed. One wonders whether this is not much more a physical picture than the strict linear confinement obtained by lattice gauge theory, the only available alternative approach for a non-perturbative description. But like the light-cone approach, the latter is burdened with its own pecularities among them the far extrapolation down to the pion.

In fact the present model runs short in several aspects. That the regulator function should apply to the whole kernel, and not only to one piece of the interaction, is a technical detail, which can be easily corrected in future work. More importent is whether the two-gluon annihilation interaction, which was omitted here on perpose, can account for the flavor diagonal mesons and the so important aspects of isospin.

Astounding is also that the present model for QCD differs from QED only by the color factor $\frac{4}{3}$ in the coupling constant. Can it be true that the essential difference between these two theories with their so distinctly different phenomenology is manifest only as large and small coupling?

Depite all of that I conclude that the light-cone approach to hadronic physics may now have a fair chance to compute the hadronic spectra and to predict the hadronic wave functions in line with a covariant theory.

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